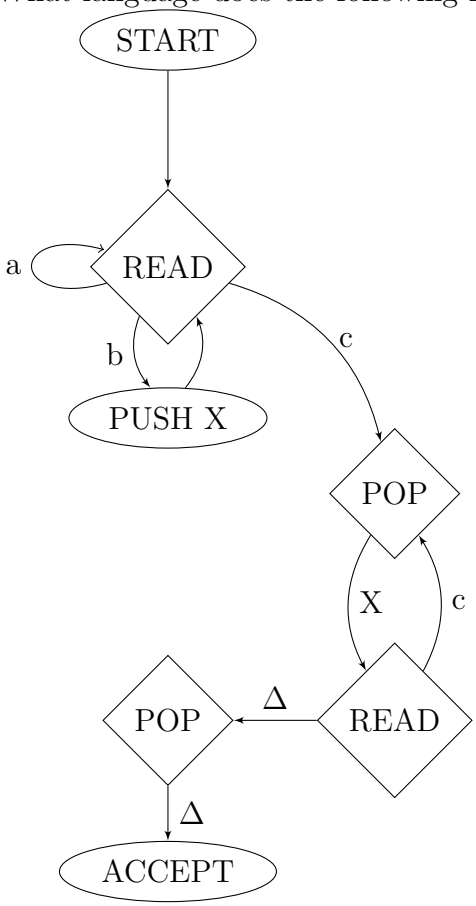
CSI 350
Practice Practice Practice

1. To start, let's define a few languages and give a few givens.
   (a) $L_1 = \{a^i b^j c^j d^i | i, j \geq 0\}$
   (b) $L_2 = \{a^i b^j q^* c^j d^i | i, j \geq 0\}$
   (c) $L_3 = \{q^* a^i b^j q^* c^j d^i q^* | i, j \geq 0\}$
   (d) $L_4 = \{q^* a^i b^j q^* c^m d^n q^* | i, j, m, n \geq 0$ and either $i = j$ or $i = m\}$
   (e) $L_5 = \{q^* a^i b^j q^* c^m d^n q^* | i, j, m, n \geq 0$, $i = j$ and $i = m\}$
   (f) $L_6 = \{q^* a^i b^j q^k q^k c^m d^n q^* | i, j, k, m, n \geq 0\}$
   (g) $L_7 = \{q^i a^j p^{2i} b^k q^i | i, j, k \geq 0\}$
   (h) L_8 is unknown to you but it has been given to be a CFL.
   (i) L_9 Not much is known about this language but...
   (j) $L_9 \cap L_8$ is not a CFL. ($\cap$ means intersection)
   (k) $L_{10} = \{q^i a^j q^{2i} b^k q^i | i, j, k \geq 0\}$
2. Write grammars for L_1, L_2 and L_3.
3. Write PDAs for L_1, L_2 and L_3
4. What language does the following grammar G_1 describe:
   S $\rightarrow$ T2 | U
   T $\rightarrow$ 0T1 | 01
   U $\rightarrow$ 0U2 | 012
5. Given the grammar G_1 from above, show the derivation of: 0001222
6. Given the grammar G_1 from above, show the parse tree of: 00112
7. Draw a PDA for the grammar G_1.
8. Is G_1 in CNF?
9. Convert G_1 into CNF.

10. What language does the following PDA describe. Draw a grammar for it.



11. Convert one of your grammars from above into Chomsky Normal Form (CNF).

12. How can you tell if a grammar is in CNF?

13. How can you tell if a language might be inherently ambiguous?

14. Are any of the languages above inherently ambiguous?

15. How can you tell if a grammar is ambiguous?

16. Are any of your grammars ambiguous? How would you tell?

17. What is the error in the following proof sketch. L_3 is not a context free language (CFL) because the string s=$a^p b^p q^p c^p d^p$ Since the number of b's in that language must be equal to the number of c's and there is no way for the pumpable parts to reach both the b's and c's, pumping up $z^{(2)}$ or down $z^{(0)}$ will not have an equal nubmer of b's and c's. Therefore it isn't a CFL.

18. What is the error in the following proof sketch (recall the givens from above)?
    (a) Assume L_9 is a CFL.
    (b) Given L_8 is a CFL.
    (c) And given $L_9 \cap L_8$ is not a CFL.
    (d) Since CFLs are closed.
    (e) Therefore, by contradiction, L_9 is not a CFL.
19. Show that CFL are closed under the Kleene operators.
20. Prove that CFLs are not closed under intersection. (Huge hint: you can do this by providing only one counter example.)
21. Prove L_7 is not a CFL.
22. Give the tightest classification of the following languages L_2, L_5, and L_6. Now, prove the tightest classfication of each.
23. What is the tightest classification of L_{10}? Can you describe the "gist" of why that is true? For a true challenge, prove it (although that is likely beyond the scope of our exam).